

The Boundary Face Method with Variable Approximation by B-spline Basis Functions

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Abstract

The objective of this work is to use the isogeometric analysis to solve the Boundary Integral Equation (BIE). The concept of isogeometric analysis[1] was firstly proposed by Hughes, and implemented in the Finite Element Method (FEM). The main idea of isogeometric analysis is that the same interpolation functions are used to construct an exact geometric model and to approximate physical variables. As a result, the exact geometry can be maintained at all levels within an adaptive analysis. There are two main aspects included in the concept, one is to build a surface representation of an exact geometry, by which the transformation from parametric coordinates to the Cartesian coordinates can be easily performed, hence the normal and tangent vectors on the surface can be obtained from CAD modeling. The other aspect is to use the same interpolation functions in the analysis that are set up by solid modeling, possibly the mesh can also be shared with the solid modeling.

As an important numerical method, Boundary Element Method (BEM) has been widely studied in the last few decades. It is based on the study of the equation governing field problems in the form of BIE rather than the more usual differential ones. One of the most important features of the BEM is that only the boundaries of the region being investigated have to be discretized, which therefore leads to much fewer discrete elements than any other method, such as the FEM in which the whole region need to be divided into many elements. Thus, a large amount of CPU time and resource used for discretization can be saved. In the application of the isogeometric analysis in the numerical solution of BIE, exact geometric model can be obtained through the Boundary representation (Brep) of any solid modeling packages.

The Boundary Face Method (BFM)[2] has been implemented with boundary variables approximated by the Moving Least Squares (MLS). Similar to the isogeometric analysis, the geometric data at Gaussian quadrature points, such as the coordinates, the Jacobian and the outward normal are calculated directly from the exact geometry rather than elements interpolation, thus the geometric error is eliminated. However, the BFM analysis is not isogeometric in a precise sense, as the parameters used for description of the boundary faces and for approximation of the field variables are different.

As a primary step, this paper presents a new implementation of BFM, in which the B-spline basis of three orders was used to approximate the boundary variables. The B-spline basis possesses useful mathematical properties, such as the ability to be refined through knot insertion, the variation diminishing, the convex hull and the compact support. As for the compact support, the basis functions are defined on the local knot vector, but the boundary variables are still defined on the global as control points. In order to ensure that the number of control points in all columns is equal to the number of bases, we use different meshes for the control net and parametric net. Numerical examples for 3D potential and elastostatic problems indicate that B-spline

basis as an interpolation function has more advantages over the MLS approximation, not only in terms of accuracy but also in terms of CPU time and convergence rate.

As a second step, the B-spline basis will be used to construct the geometric model to achieve a truly isogeometric analysis. The higher precision and tighter integration of the overall modeling-analysis process are expected. Finally, a new method of T-spline[3] was proposed by T.W. Sederberg is also planned to be implemented in the BFM. The control points and knot vectors in T-spline are defined on the local, which allows more freedom to choose the B-spline basis functions. Using T-spline for isogeometric analysis will be a direction of our future work.

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